Sample Size for Descriptive Studies

Sample Group	Continuous Outcome (Sample Mean)	Dichotomous Outcome (Sample Proportion)
One Sample	$n = \left(\dfrac{Z\sigma}{E} \right)^2$	$n = p\left(1 - p\right)\left(\dfrac{Z}{E} \right)^2$
Two Independent Samples	$n_i = 2\left(\dfrac{Z\sigma}{E} \right)^2$	$n_i = \left\{ p_1(1 - p_1) + p_2(1 - p_2) \right\}\left(\dfrac{Z}{E} \right)^2$
Two Matched Samples	$n = \left(\dfrac{Z\sigma_d}{E} \right)^2$	$n = p\left(1 - p\right)\left(\dfrac{Z}{E} \right)^2$

If population is small, use correction: $\mathbf{n_c}$ = Nn/ (N+n-1) *(reduces slightly)*

© Grace Perez

Estimation for Comparative Studies

Sample Group	Continuous Outcome (Sample Mean)	Dichotomous Outcome (Sample Proportion)
One Sample	$n = \left(\dfrac{z_{1-\alpha/2} + z_{1-\beta}}{\delta}\right)^2,\quad \delta = d = \dfrac{\lvert \mu_0 - \mu_1 \rvert}{\sigma}$ $\hat{\sigma} = \text{SD}$	$n = \left(\dfrac{z_{1-\alpha/2} + z_{1-\beta}}{\delta}\right)^2,\quad \delta = d = \dfrac{\lvert p_1 - p_0 \rvert}{\sigma}$ $\hat{\sigma} = \sqrt{(p_1)(1-p_1)}$
Two Independent Samples*	$n_{i\ per\ group} = 2\left(\dfrac{z_{1-\alpha/2} + z_{1-\beta}}{\delta}\right)^2,\quad \delta = d = \dfrac{\lvert \mu_1 - \mu_2 \rvert}{\sigma}$ $\hat{\sigma} = Pooled\ SD = \sqrt{\dfrac{(SD_1{}^2 + SD_2{}^2)}{2}}$	$n_{i\ per\ group} = 2\left(\dfrac{z_{1-\alpha/2} + z_{1-\beta}}{\delta}\right)^2,\quad \delta = d = \dfrac{\lvert p_1 - p_2 \rvert}{\sigma}$ $\hat{\sigma} = \sqrt{(\bar{p})(1-\bar{p})}$ $\bar{p} = (p_1 + p_2)/2$
Two Matched Samples	$n = \left(\dfrac{z_{1-\alpha/2} + z_{1-\beta}}{\delta}\right)^2,\quad \delta = d = \dfrac{\lvert \bar{D} \rvert}{\sigma}$ $\hat{\sigma} = \sigma_D$ $\bar{D} = (\mu_1 - \mu_2)$	$n = \left(\dfrac{z_{1-\alpha/2} + z_{1-\beta}}{\delta}\right)^2,\quad \delta = d = \dfrac{\lvert p_1 - p_2 \rvert}{\sigma}$ $\hat{\sigma} = \sqrt{(p)(1-p)}$

*For unequal sample size per group (r:1 ratio), replace 2 with (r+1)/r to get ***n*** for smaller group; when r=1, then sample sizes are equal.

Comparing 3 or More Groups

- Independent Means (ANOVA)
  - Choose two most important pair of means you really care about
  - Calculate as for two independent samples
  - Resulting n is per group

- Repeated Measures Means
  - Choose two most important pair of means (e.g., baseline and last)
  - Calculate as for two matched samples
  - Resulting n is per group (if there are several groups)

- Independent Proportions (Categorical)
  - Choose two most key groups; collapse outcome to binary (e.g. worst/best)
  - Calculate as for two groups
  - Resulting n is per group

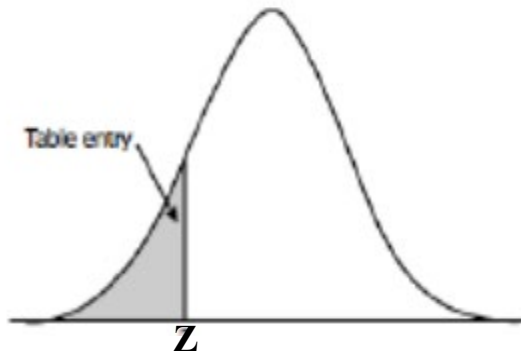 *Note:* It's always a good idea to consult a statistician

Standard Normal Table

Table entry for Z is the area under the curve (AUC) to the left of Z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Table

Table entry for Z is the area under the curve (AUC) to the left of Z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Power Analysis

- ## A priori
  - Find sample size - given effect size, α-level, and power
  - Beginning of research

- ## A posteriori
  - Find power - given sample size, effect size, and α-level
  - End of research (*post hoc power analysis*) – generally useless!
  - It tells you nothing that you do not already know from the *p* value

Power Analysis

- **General formula**

$$Z_{power} = \frac{(\text{difference})}{\text{SE(difference)}} - Z_{1-\alpha/2}$$

$$\text{Power} = AUC \text{ of Normal Prob Distributin } (Z_{power})$$

For difference in means:

$$s_{\overline{X}_1 - \overline{X}_2} = \sqrt{s^2_{pooled}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$S^2_{pooled} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}$$

For unequal samples:

$$S^2_{pooled} = \sqrt{\frac{\hat{\sigma}^2}{n_1} + \frac{\hat{\sigma}^2}{rn_1}} = \sqrt{\frac{(r+1)\hat{\sigma}^2}{rn_1}}$$

For difference in proportions:

$$\text{SE}_{\hat{p}_1 - \hat{p}_2} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\hat{p} = \frac{n_1(p_1) + n_2(p_2)}{n_1 + n_2}$$

Power Analysis

- **Standard Normal Distribution**
  - Power = area under the curve for $Z_{1-\beta}$
  - Beginning of research
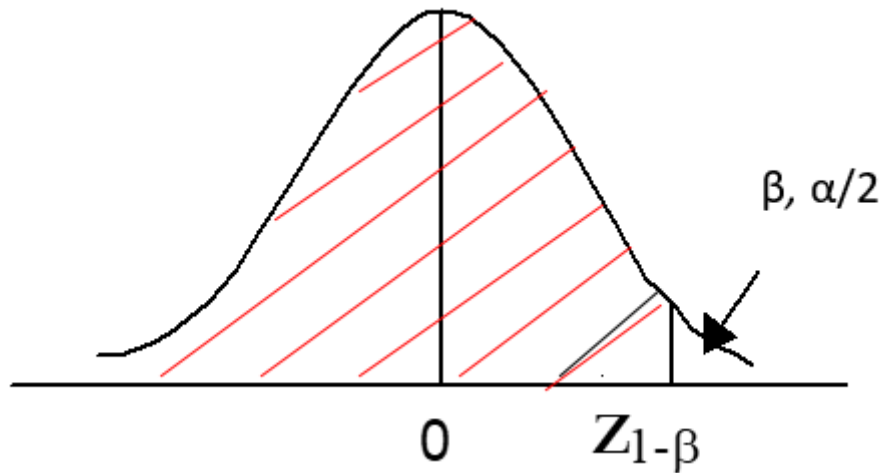
Probability (AUC)	Error Prob (β, α)	Normal Values	
		One-tailed ($Z_{1-\beta}$, $Z_{1-\alpha}$)	Two-tailed ($Z_{1-\alpha/2}$)
80%	20%	0.84	1.28
85%	15%	1.03	1.44
90%	10%	1.28	1.65
95%	5%	1.65	1.96
99%	1%	2.33	2.58

WORKED EXAMPLES

Examples

1) *We want to study the prevalence of dysfunctional breathing among asthma patients at Foothills Hospital using a survey. How many patients do we need if we want our estimate to be within 5% of the known prevalence of 30%?*

2) *A doctor wants to estimate the mean SBP in children with congenital heart disease between ages 6 and 12. How many children will she enroll if she wants her estimate to have 95% confidence, with a margin of error of 5 mmHg? From past studies, the SD for SBP in children ranges from 15-20.*

3) *A recent report from the Framingham Heart Study indicated that 26% without CHD have elevated LDL levels (>159mg/DL). A researcher believes it is higher. How many patients does he need if he wants to test if this true, with 90% power to detect a 5% difference in the proportion with high LDL?*

4) *A researcher is planning a clinical trial to evaluate if a new drug designed to lower SBP is effective compared to a placebo. How many participants does he need if he wants to show that the new drug works? He wants to see a difference of 10mmHg in SBP with power of 90%.*

Example 1

Sample size to estimate a one-sample proportion

- *What is the prevalence of dysfunctional breathing among adult patients with asthma being treated at the Foothills Hospital?*

- *We want to study the **prevalence of dysfunctional breathing** among asthma patients at Foothills Hospital using a questionnaire survey*
    - *Outcome = dysfunctional breathing (Y/N)*
    - *'Best guess' of expected prevalence (p) = 30% (0.30)*
    - *Desired width of 95% CI = 10% (i.e. +/- 5%,   25% - 35%)*

- Formula :

$$n = p\left(1 - p\right)\left(\frac{Z}{E}\right)^2 = (0.3)*(0.7)*(2^2) / (0.05)^2 = 336$$

- where $Z_{1-\alpha 1/2}$ = 1.96 ⮕ 2
- *p* = the expected proportion - here 0.30
- *E* = margin of error - here 0.05

Example 1 *continued*

Sample size to estimate a one-sample proportion

- Write up for grant or research proposal:

  "A sample of 336 adult patients with asthma will be required to obtain a 95% confidence interval of +/- 5% around a prevalence estimate of 30%. To allow for an expected 70% response rate to the questionnaire, a total of 500 questionnaires will be provided."

- **Note:**

  - The formula used is based on 'normal approximation methods', and, should not be applied when estimating proportions which are close to 0.0 or 0.1. In these circumstances 'exact methods' should be used.

  - When conducting surveys, allow for non-responses and attrition

  - Check answer using https://select-statistics.co.uk & https://www.stat.ubc.ca/~rollin/stats/ssize/

  - Compare above results with Qualtrics: https://www.qualtrics.com/blog/calculating-sample-size/

Example 2

Sample size to estimate a one-sample mean

- *What is the average blood pressure (SBP) reading in children with congenital heart disease?*

- *A doctor wants to estimate the mean SBP in children with congenital heart disease between ages 6 and 12. How many children will she enroll if she wants her estimate to have 95% confidence, with a margin of error of 5 mmHg? From past studies, the SD for SBP in children ranges from 15-20.*

  - *Outcome = SBP (mmHg)*

  - *'Best guess' standard deviation (SD) = 20   (err on the side of large!)*

  - *Desired width of 95% CI = 10 mmHg (i.e. +/- 5 mmHg)*

- Formula:

$$n = \left(\frac{Z\sigma}{E}\right)^2 \quad = (20*2)^2 / (5)^2 \quad = 64 \quad \textit{(Always Round Up)}$$

  - where $Z_{1-\alpha1/2}$ = 1.96 ⮕ 2

  - *SD* = 20 mmHg

  - *E* = margin of error - here 5 mmg

Example 2 *continued*

Sample size to estimate a one-sample mean

- Write up for grant or research proposal:

    "A sample of 64 children with congenital heart disease will be required to estimate the mean SBP, with 95% confidence interval of +/- 5mmHg. Previous studies have shown the variation in SBP as SD=15-20 mmHg. We chose the larger SD in order to produce a confidence interval with a more prudent margin of error."

- **Note:**

    - Had we used SD=15, we would have n=36. Because estimates of SD were derived from studies of children with heart defects (quite rare so very variable), it would be advisable to use the larger SD and plan to enroll 64 patients.

    - Selecting a smaller sample size could potentially produce a confidence interval estimate with a larger margin of error

    - Check answer at https://select-statistics.co.uk & https://www.stat.ubc.ca/~rollin/stats/ssize/

Example 3

Sample size to compare 2 group proportions

- *Does a daily high-dose supplement of Vitamin D+Calcium reduce risk of bone fractures?*

- *In Canada, osteoporosis affects approximately 1.5 million Canadians over 40 years old (10%) , with 1 in 5 men and women having had a bone fracture. We plan to conduct a trial with high dose supplement of Vit D + Ca versus placebo control on men and women with osteoporosis over 5 years. How much sample do we need if we wanted to detect a reduction in bone fracture risk to 1 in 7?*

  - <u>Group 1</u>: Placebo group expected fracture rate = 1 in 5 (0.21)
  - <u>Group 2</u>: Vit D+Ca group expected 7% risk reduction (1 in 7 = 0.14)
  - Find sample size if α=0.05 for both 80% and 90% power
  - $p_1 - p_2$ = .07 ; $z_{1-\alpha/2}$ = 1.96 ⮕2 $z_{0.80}$=0.84 , $z_{0.90}$ = 1.28

- Formula to compare 2 proportions

$$\frac{2(\bar{p})(1-\bar{p})\left(z_{1-\alpha/2} + z_{1-\beta}\right)^2}{(p_1 - p_2)^2}$$ = 2 {(0.175)(1-0.825)} *(2 + 0.84)^2 / (.07) ^{2} = 480/group

$$\bar{p} = (p_1 + p_2)/2$$ = (0.21+0.14)/2 = 0.175

- Power = 80% : n per arm = 480 ; total n = 960 patients with osteoporosis
- Power = 90% : n per arm = 635 ; total = 1,270 patients with osteoporosis

Example 3 *continued*

Sample size to compare 2 group proportions

- Write up for grant or research proposal:

    "In order to compare the risks of bone fracture between the Vit D + calcium treatment group with the placebo group after 5 years, we need a sample of 1,270 patients with osteoporosis (635 per arm). This sample size is sufficient to detect a risk reduction of 7% if it exists, with a power of 90%. We expect a drop-out rate of 20% so we will enroll a total 1,600 to allow for dropouts."

- **Note:**

    - Number to enroll * %retained = desired sample size, n

    - Number to enroll = desired sample size / %retained;   1,270/0.80 = 1600 total

    - Check answer at https://select-statistics.co.uk & https://www.stat.ubc.ca/~rollin/stats/ssize/

Example 4

Sample size to compare 2 group proportions (**unequal**)

- *Does a daily high-dose supplement of Vitamin D+Calcium reduce risk of bone fractures?*

- *Suppose that the researchers want the treatment group to have <u>twice the sample size</u> of the control (placebo) group. How much sample do we need if we wanted to detect a reduction in bone fracture risk to 1 in 7?*

  - <u>*Group 1*</u>: Placebo group facture rate = 0.21
  - <u>*Group 2*</u>: Vit D+Ca group fracture = 0.14
  - Find sample size if $\alpha=0.05$ for both 80% and 90% power; **ratio=2:1 or r=2**
  - $p_1 - p_2 = .07$; $z_{1-\alpha/2} = 1.96$ ▢2 $z_{0.80}=0.84$, $z_{0.90} = 1.28$

- Formula for 2 proportions with unequal sample size:

$$\frac{(r+1)}{r} \frac{(\bar{p})(1-\bar{p})\left(z_{1-\alpha/2} + z_{1-\beta}\right)^2}{(p_1 - p_2)^2} = (3/2)\{(0.175)(1-0.825)\}*(2+0.84)^2/(.07)^2 = 360 \text{ for smaller group}$$

- Power = 80% : treatment arm = 720; control arm = 360; total n = 1080 patients

- Power = 90% : treatment arm = 954 ; control arm = 477; total n = 1431 patients

© Grace Perez

Example 4 *continued*

Sample size to compare 2 group proportions (**unequal**)

- Write up for grant or research proposal:

  "In order to compare the risks of bone fracture between the Vit D + calcium treatment group with the placebo group after 5 years, we need a sample of 954 patients with osteoporosis in the treatment group, and 477 in the placebo group. This sample size is sufficient to detect a risk reduction of 7% if it exists, with a power of 90%. To allow for an expected 20% drop-out rate, we will enroll 1200 in the treatment group and 600 in the control group, for a total of 1,800."

- **Note1:**
  - Number to enroll * %retained = desired sample size, n
  - Number to enroll = desired sample size / %retained;
  - 954 /0.80 =1193 for treatment arm ; 477 /0.80 = 597 for control arm
  - Total n = 1,790
  - Can round up to 1200 and 600 in treatment and control, respectively.
- **Note2:**
  - For online calculators, multiply ***n*** by 0.75 to get smaller group n; 1.5 for bigger group n.
  https://select-statistics.co.uk & https://www.stat.ubc.ca/~rollin/stats/ssize/
  - Adjust for drop-outs accordingly

Example 5

Sample size to compare 2 group means

- *Is there is a difference between 2 types breast carcinoma patients (invasive ductal vs non-invasive ductal) in terms of mean CA153 tumor marker levels?*

- *How many cases would be needed if the researchers want to prove that there is a difference of at least 20 U/ml. Previous studies have shown non-invasive ductal Ca patients have mean±SD: 64±52 and invasive patients have mean±SD: 86±49?*

  - *Group 1*: Invasive Breast Ca - mean±SD: 86±49 U/ml

  - *Group 2*: Non-invasive Breast Ca - mean±SD: 64±52 U/ml

  - Find sample size if α=0.05 for both 80% and 90% power

  - $\mu_1 - \mu_2$ = 20 ; $z_{1-\alpha/2}$ = 1.96 ⬚2 $z_{0.80}$=0.84 , $z_{0.90}$ = 1.28

- Formula

$$\frac{2\sigma^2\left(z_{1-\alpha/2} + z_{1-\beta}\right)^2}{(\mu_1 - \mu_2)^2}$$ = 2 (2500) *(2 + 0.84)²/(20)² = 101/group

$$Pooled\ SD = \sqrt{\frac{(SD_1^2 + SD_2^2)}{2}}$$ = √(52²+ 49²)/2 ⬚ 50

- Power = 80% : n per arm = 101 ; total n = 202 patients
- Power = 90% : n per arm = 135 ; total n = 270 patients

Example 5 *continued*

Sample size to compare 2 group means

- Write up for grant or research proposal:

    "In order to evaluate a potential difference in mean CA153 tumor marker levels for invasive ductal carcinoma and non-invasive ductal carcinoma, we need a sample of 270 patients (135 per arm). This sample size is sufficient to detect a difference of 20 U/ml if it exists, with a power of 90%. To allow for attrition rate of 10% (inadequate blood samples, missed appointments, etc), so we will enroll a total 300 patients."

- **Note:**

    - Number to enroll * %retained = desired sample size, n
    - Number to enroll = desired sample size / %retained;   270/0.90 = 300 total
    - Check answer at https://select-statistics.co.uk & https://www.stat.ubc.ca/~rollin/stats/ssize/

Example 6

Sample size to compare means from matched samples

- *Does a lifestyle program (diet + exercise for 3 months) lead to improved SBP and BMI measurements, compared to baseline?*

- *Suppose an investigator wishes to determine sample size to detect a 5 mmHg change in SBP and 1 unit in BMI from baseline, after 3 months of lifestyle intervention. From literature, change in SBP has SD 18-20, while change in BMI has SD between 4-6 units.*

  - <u>Outcome 1</u>: SBP, SD = 18-20 mmHg

  - <u>Outcome 2</u>: BMI, SD = 4-6 units

  - Find sample size if α=0.05 at 80% power ($z_{1-\alpha/2}$=1.96 ⏀2, $z_{0.80}$=0.84, $z_{0.90}$=1.28)

- Formula

$$\frac{\sigma_D^2 \left(z_{1-\alpha/2} + z_{1-\beta}\right)^2}{\overline{D}^2}$$

     1) $(20^2)*(2+0.84)^2/(5^2)$ = **130 participants for SBP**

     2) $(6^2)*(2+0.84)^2/(1^2)$ = **290 participants for BMI**

- For SBP outcome: n = 130 participants
- For BMI outcome: n = 290 participants
- Use the bigger sample size! Round up to 300 to improve power!

Example 6 *continued*

Sample size to compare means for matched samples

- Write up for grant or research proposal:

  "A sample of 300 people will be invited to participate in a lifestyle intervention (diet+exercise) program for 3 months. This sample size is sufficient to detect changes from baseline measurements of 5 mmHg in SBP and 1 unit in BMI, with a power of 80%."

- **Note:**

  - You can also compute the sample size for power = 90% and show in your grant the different options.

  - Check answer at https://www.stat.ubc.ca/~rollin/stats/ssize/ and http://www.sample-size.net/sample-size-study-paired-t-test/

  - If you will use Rollin Brant's calculator, use calculator for single population mean

Example 7

Power calculation

- *Are male nurses more emphatic than female nurses?*
- *Suppose you wish to compare Empathy Quotient scores (EQ) between male and female nurses. You want to calculate how much power you will have to see a difference of 3.0 points between two groups: 30 male nurses and 30 female nurses. If the SD is expected to be about 10 on an EQ tool for both groups, <u>what is the power of your study</u>?*
  - <u>Outcome</u>: EQ score, SD = 10 points
  - Find power if α=0.05 and n=30/group

- Formula

$$Z_{power} = \frac{(\text{difference})}{SE(\text{difference})} - Z_{1-\alpha/2}$$

$$= (3)/2.582 - 1.96 = \textbf{-0.797}$$

$$SE(\text{diff}) = \sqrt{\frac{10^2}{30} + \frac{10^2}{30}} = 2.582$$

$$\text{Cohen's d} = \frac{|\mu_1 - \mu_2|}{\sigma} = 0.30$$

- $\mathbf{Z_{power}}$ **= -0.797 ;** Prob(Z ≤ -0.79) =.21, **We have only 21% power**

Example 7 *continued*

Power calculation

- Check answer at https://www.stat.ubc.ca/~rollin/stats/ssize/ & https://www.danielsoper.com/statcalc/

Inference for Means: Comparing Two Independent Samples

(To use this page, your browser must recognize JavaScript.)

Choose which calculation you desire, enter the relevant population values for mu1 (mean of population 1), mu2 (mean of population 2), and sigma (common standard deviation) and, if calculating power, a sample size (assumed the same for each sample). You may also modify α (type I error rate) and the power, if relevant. After making your entries, hit the calculate button at the bottom.

- ○ Calculate Sample Size (for specified Power)
- ◉ Calculate Power (for specified Sample Size)

Enter a value for mu1: `30`
Enter a value for mu2: `33`
Enter a value for sigma: `10`

- ○ 1 Sided Test
- ◉ 2 Sided Test

Enter a value for α (default is .05): `.05`
Enter a value for desired power (default is .80): `0.21`
The sample size (for each sample separately) is: `30`

[Calculate]

Reference: The calculations are the customary ones based on normal distributions. See for example *Hypothesis Testing: Two-Sample Inference - Estimation of Sample Size and Power for Comparing Two Means* in Bernard Rosner's **Fundamentals of Biostatistics**.

Rollin Brant
Email me at: rollin@stat.ubc.ca

Observed effect size (Cohen's d): `0.3`
Probability level: `0.05`
Sample size: `60`

[Calculate!]

Observed power (one-tailed hypothesis): 0.30960711
Observed power (two-tailed hypothesis): 0.20625583

Output from www.danielsoper.com/statcalc/ calculator

Example 7 *continued*

Power calculation

- *Are male nurses more emphatic than female nurses?*

- *How many people would you need to sample in each group to achieve power of 80% (corresponds to $Z_{1-\beta}=.84$) to see a difference of 3.0 points between two groups?*

  - *Outcome*: EQ score, SD = 10 points

  - Find sample size if $\alpha=0.05$ at 80% power ($z_{1-\alpha/2}=1.96 \approx 2$, $z_{0.80}=0.84$, $z_{0.90}=1.28$)

- Formula

$$\frac{2\sigma^2\left(z_{1-\alpha/2} + z_{1-\beta}\right)^2}{(\mu_1 - \mu_2)^2} \quad = 2\,(100)*(2 + 0.84)^2/(3)^2 \quad = \;180/\text{group}$$

- Power = 80% : n per group = 180 ; total n = 360

Notes for Power

- For practical reasons, it's useful to try to calculate the power of your study *before* conducting it

- If power is very low, there's no point in conducting the study

- Basically, you want to make sure you have a reasonable shot at finding an effect (if it exists) - which is why grant reviewers want them!

- For grant applications, it's a good idea to show the power for a variety of sample sizes and effect sizes in a table so that the grant reviewers can see that there is sufficient power to detect a range of effect sizes with the proposed sample size